

Least-time Path Algorithm Based on Missile Guidance

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Abstract: A dynamic and real-time single-vehicle path algorithm considering system optimization is proposed by the method of simulating missile guidance. Targeted at minimizing the run-time of a single vehicle, the algorithm plans a path for the vehicle to avoid traffic congestion and help optimize the overall traffic state. This algorithm begins with preliminary path planning to find the points from which ideal paths that help balance the vehicle traffic flow can be generated. Then the algorithm plans for the ideal paths before the actual path is derived from the ideal ones. Throughout the driving process, the algorithm dynamically and cyclically plans the path in real-time, and dynamically revises the plan of the path covering the sections ahead based on the traffic state. Simulation showed that the proposed algorithm effectively enables vehicles to avoid congestion and save travel time, and helps balance the vehicle traffic flow and optimize the traffic state.

Key Words: traffic engineering; path algorithm; simulating missile guidance; system optimization; ideal path

1 Introduction

Classical path algorithms, such as the Dijkstra algorithm^[1,2] and A* algorithm^[3,4] were initially aimed at generating the shortest path. Then some researchers applied least-time algorithms to accommodate for the dynamic characteristics of traffic systems. Typical algorithms for time-dependent vehicle path planning problems include the restricted dynamic programming heuristic algorithm^[5], genetic algorithm^[6], ant colony optimization^[7], and simulated annealing^[8].

The above-mentioned algorithms mainly targeted at optimizing the path of a single vehicle as expected by the driver, who, as a user, whose top priority is the shortest path or the least travel time^[9]. Meanwhile, the intent of traffic manager is the optimal traffic system in which the distribution of traffic flow is balanced and the overall delay is minimal. If these algorithms are simultaneously used by multiple vehicles in a same road network, they would be unfavorable for the optimization of traffic system and traffic jam occurs^[10]. The path algorithms proposed in reference [11] consider time window constraint, and some of them take into account the optimization of traffic systems. However, if there are too many vehicles, excessively huge calculation is required and inter-objective conflicts may occur, which potentially prevents the system from reaching an optimal solution. In addition, a

traditional dynamic single-vehicle path algorithm need to predict the state of traffic flow in a road network^[12] before it can dynamically produce a path for the vehicle. Therefore, the planning accuracy relies on how the state is predicted in real-time. High precision of the predicted state inevitably requires a large number of calculations in the prediction and planning. This is why such algorithms are unsuitable for multiple vehicles' path determination in a large-scale traffic network.

To this end, a dynamic and real-time single-vehicle path algorithm considering system optimization is proposed. The proposed algorithm, which has already been applied to simulate missile guidance^[13], is able to consider the requirements of both the driver and the traffic manager^[14]. It meets the driver's objective that he/she should be guided to avoid congested road sections and save travel time. To attain an ideal path, the proposed algorithm examines dynamic change to the congestion across a traffic network and finds the points of the ideal paths that help balancing the vehicle traffic flow. Then the algorithm considers a trade-off between the ideal path and the path that meets the objective of a single vehicle, to generate the final path to be executed. The algorithm cyclically plans the path while the vehicle travels. In other words, it dynamically revises the path in remain sections based on the real-time information of traffic state

provided by an information platform^[14].

2 Dynamic path algorithm simulating missile guidance

The algorithm enables a single vehicle to avoid congested road sections and minimize its travel time; it also generates the paths which favor traffic flow balance and system optimization.

2.1 Principle and procedure of the algorithm

According to reference [15], the problem of precise missile guidance is how to strike (1) as close as possible to the target and (2) from a specific direction. It is because that an ideal route for the missile is usually the path with minimal travel time. However, a missile cannot always fly along an ideal flying route due to the effect of gravity. To overcome the effect of gravity or other spatial states, the route of the missile in the air may be too long or require much more travel time. The influence from gravity is the fundamental cause of the conflict in missile guidance. Therefore, the route generated by missile guidance is a trade-off between the ideal route and the route under certain spatial state.

Vehicle path planning holds some similar features with missile guidance such as: (1) the final objective—to arrive at a destination using minimal travel time; (2) they all need to overcome the resistance from some influential factors like the influence from traffic state to vehicles, particularly in traffic congestions. Due to the effect of the traffic state, the path-planning may cause traffic congestion if all the vehicles only try to meet their individual objectives but ignore the efficiency of the entire traffic system. However, if the system optimization becomes the first concern, i.e., to balance the traffic state, the planned paths may introduce too much detour or require too much travel time. Therefore, missile guidance is a good way to settle the conflicts between the two objectives.

Vehicle path planning is distinct from the missile guidance in that: (1) traffic state of a traffic system varies over time and place, but for missile guidance, the gravity is constant for any spatial environment; (2) the former problem is restricted by the structure of road network, traffic speed limitation, and the scope of path planning, while the latter one is free from such restrictions in spatial environment.

To address these two differences, the state of a traffic system, which corresponding to the gravity item in missile guidance, should be expressed in real-time. And then the ideal paths for a single vehicle which is beneficial for system optimization should be planned with missile guidance simulation. At last, the actual structure of road network, speed limit, and scope of path planning are considered in order to generate an actual path that is a trade-off between the single-vehicle path-planning and the system optimization.

The fundamental principles are illustrated as follows:

First, we need to determine the ideal paths for the single

vehicle. An ideal path is the theoretically best one that enables the vehicle to avoid congested road sections and minimize travel time without considering the restrictions in a real traffic network.

Fig. 1 elaborates such principle of the algorithm. Assuming that heavy traffic flow occurs at crossings a , b , e , and f , as well as the sections connected to them; light flow occurs at crossings c , d , and g , and any other sections. The ideal path from origin A to destination B is a curve that can avoid heavy-traffic crossings and sections between the two points. If we suppose that all crossings are connected to each, and path $AdcB$ is observed to be the best one appropriating the ideal path and meets the objective of avoiding congested road sections.

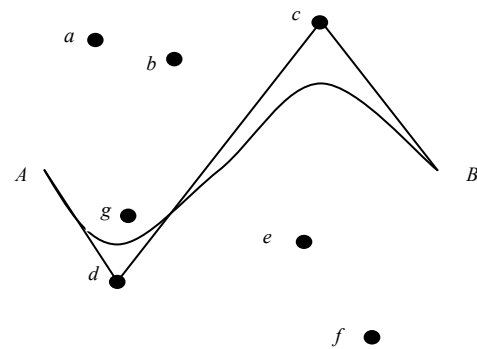


Fig. 1 Principle of the algorithm

The path algorithm is operated as follows:

Step 1: The preliminary path planning is actually an operation to identify the points of ideal paths generated (namely, the "IPG points"). It evaluates real-time traffic flow and congestions in traffic system and marked out the smooth points that can achieve the goal of system optimization with balanced traffic flow.

Step 2: Using the motion equation to test whether the IPG points meet the requirements of path planning and match the actual traffic status. Then connect all the satisfactory IPG points to form an ideal path.

Step 3: Taking the ideal path as the center and searching by a fixed step along the Y-axis to find an actual path that meets the objective of path planning.

Step 4: Throughout the driving process, we define the start point of the road section as the vehicle origin and the terminal of the road as the destination, and then dynamically evaluate paths according to the real-time traffic state. The planning is conducted one more time when the vehicle arrives at the next road section. Such cyclic planning will be carried on until the vehicle reaches its destination and forms a real-time dynamic algorithm.

2.2 Algorithm and steps of preliminary path planning

The preliminary path planning requires IPG points to avoid congested road sections and get as close as to smooth sections.

The road network is regarded as a "gravitational field" in respect of the congestion state in traffic system (the field corresponds to the spatial gravitational field in missile guidance). This step is also defined as a procedure to find the IPG points. The major purpose is to indicate the real-time traffic state (corresponding to the gravity item in missile guidance) of the points of ideal paths generated, according to traffic volume and congested status of traffic system. The IPG points have smooth traffic and also helpful for system optimization.

The steps are summarized as follows:

Step 1: Setting a coordinate system with the city center as the origin and the longitude and latitude directions as x and y axes, and then indicating intersections and traffic lighting according to the actual location and real-time traffic state.

Step 2: Marking out the origin and destination of the vehicle in the road section in real-time. The section between the two points along x -axis is defined as the region for preliminary path planning.

Step 3: Distinguishing the IPG points in preliminary path planning region and the solving process is described below:

Fig. 2 indicates the IPG points. The coordinates of z —destination of path planning, are fixed and denoted as (x_z, y_z) . All the points excluding the origin and destination are sorted in the ascending order of their x -coordinate values. The points with the same x -coordinate value are grouped in a set. It should be noted that, for the purpose of path planning, the points on different lanes of the same section are regarded as different point sets and should be calculated separately. The points in set F are written as $f_e, e=1, 2, \dots, n$. m_{f_e} , the degree of congestion corresponding to each point, is proportional to the time taken to pass through the section, which includes both the time running through certain lanes and passing intersections. The passing time is reversely proportional to the length of the section.

It is assumed point f_e is located on lane l or intersection c on the road section.

$$m_{f_e} = (m_c T_c + l_l / v_l) / L_l \quad (1)$$

where, m_c is the degree of congestion at intersection c ; T_c is the signal cycle at intersection c ; l_l is the number of vehicles across the entire length of lane l ; v_l is the traffic speed of lane l ; L_l is the length of lane l .

Coordinates of each point in the set is represented as (x_f, y_{f_e}) . IPG point f has the same coordinates on x -coordinate with these points and is written as x_f , and the y -coordinate is $y_f(t)$

$$y_f(t) = \sum_{e=1}^n (y_{f_e} / m_{f_e}) / \sum_{e=1}^n (1 / m_{f_e}), f=1, 2, \dots, n \quad (2)$$

The ideal degree of congestion at the IPG point (m_f) is

$$m_f = \sum_{e=1}^n m_{f_e} y_{f_e} / \sum_{e=1}^n y_{f_e}, f=1, 2, \dots, n \quad (3)$$

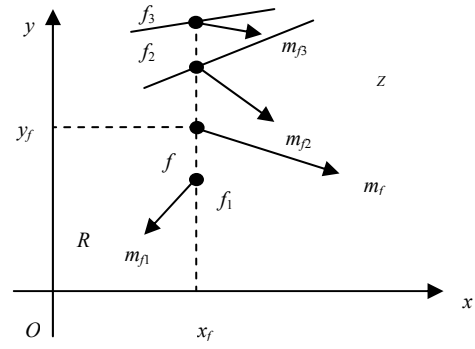


Fig. 2 Schematic diagram of IPG points

2.3 Algorithm and steps of ideal path planning

Referring to the definition of the ideal path, we should take all the IPG points (the points with smooth traffic state and also helpful for system optimization) as the candidate points of ideal path. Reference [16] addressed that the guidance algorithm is suitable for the cases with (1) smooth trajectory, (2) limited acceleration, and (3) high shock angle. This is because that missile guidance requires the missile to fly along a smooth curve or with steady velocity and dynamic performance. Under the real traffic circumstances, the road network is always fixed and vehicles are not required to run along a route of "smooth curve". Each selected point on the ideal path should be "relatively continuous" with the points selected before and after it in terms of the space and time. It means that the ideal points should spatially meet the requirements for path planning and temporally matches the actual traffic requirements. To examine if an IPG point is satisfactory, the motion equation formulated for the vehicle reaching its destination, similar with the equation of motion for missile guidance. An ideal path is worked out by linking all the satisfactory IPG points from the origin of to the destination.

Step 1: Forming the motion equation for the vehicle to reach its destination by simulating the equation of motion of missile guidance. As the IPG points are "discrete", satisfactory IPG points are chosen to ensure the "continuity" of the ideal path.

Fig. 3 shows the scheme of the motion equation for the vehicle to reach its destination. The coordinate system oxy is the urban traffic coordinate system and destination z is fixed. If we assume that the vehicle locates at the IPG point j with the velocity of v_f , then $R_f(t)$, the distance that the vehicle is away from the destination z is

$$R_f(t) = \sqrt{(x_z - x_f)^2 + (y_z - y_f(t))^2} \quad (4)$$

where, θ is the angle between the velocity vector of the vehicle and axis ox ; and ϕ is the angle between the line linking the vehicle to the destination and axis ox ; and β is the angle between the velocity vector of the vehicle and the line linking the vehicle to the destination.

The motion equation for the vehicle to reach its destination is formulated as follows

$$dR_f(t)/dt = -v_f \cos \beta \quad (5)$$

$$g'(x_f) = bm_f = -v_f \sin \theta / t = -v_f d\theta / dt \quad (6)$$

For the purpose of path planning, v_f works to avoid congestion, and $g'(x_f)$ is proportional to m_f —the ideal degree of congestion at the IPG point.

The geometric relationship between the angles is depicted as follows

$$\phi = \beta + \theta \quad (7)$$

$$\tan \phi = (y_z - y_f(t)) / (x_z - x_f) \quad (8)$$

Combining the two equations into an equation set, we get v_f , ϕ , θ , and β . v_f should meet the requirements of road speed limit. The relationship among ϕ , θ , and β should meet the requirements of the path planning.

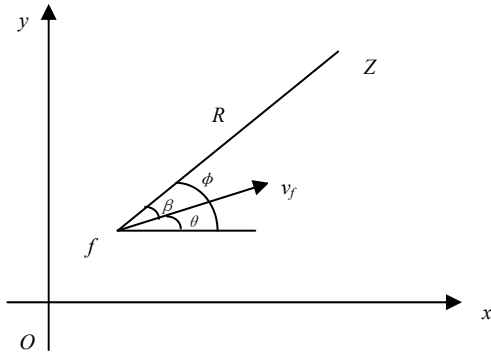


Fig. 3 Equation of motion

Step 2: Determining if the IPG points meet the requirements of path planning and actual traffic situations.

As the road network is given, if we do not take into account the route coverage, the calculated path would be too long or with too many detours. If so, the acceptance rate of guidance would be less. Hence, to meet the driver's requirements, the scope of path planning should be a square with the line connecting the origin and destination as the diagonal, or a circle with the line as the diameter, or a square with the line as the central line.

Fig. 4 plots out the scope for path planning. When f is an IPG point, j is the origin of path planning, and z is the destination of path planning.

If the path is determined in a square region, the angle should range

$$\begin{cases} \alpha_1 < 45^\circ \\ \alpha_2 < 45^\circ \end{cases} \quad (9)$$

where, α_1 and α_2 are the angles by which the line linking the IPG point and the crossing is away from the line linking the origin and destination.

If the path is planned in a circle region, the angle should meet

$$\alpha_1 + \alpha_2 < 90^\circ \quad (10)$$

If the path is planned in a square region, the angle should be situated in

$$\begin{cases} \alpha_1 < 90^\circ \\ \alpha_2 < 90^\circ \\ D_{jz} < L_{jz} / 2 \end{cases} \quad (11)$$

where, D_{jz} is the distance between points f to j and z ; and L_{jz} is the distance between points j and point z .

In the subsequent simulation, we adopt a circle region for path planning.

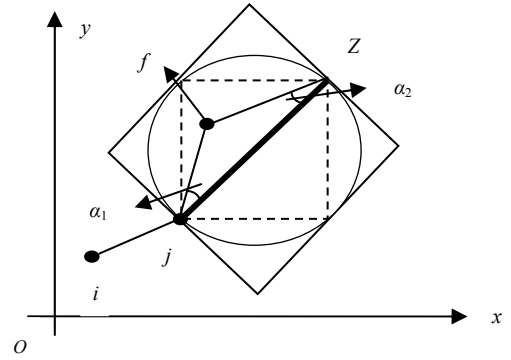


Fig. 4 Scope of path planning

The actual traffic requirements are primarily the speed limit along road sections and across intersections. An IPG point is retained when the motion equation can generate a satisfactory solution meeting the requirements of value range and speed limit; otherwise, it should be removed.

Step 3: After getting all IPG points that meet the requirements of path planning and the requirements of real traffic, we connect all these points in the ascending order along the x -axis to obtain the ideal paths.

2.4 Selection criteria of actual path

The guidance law description is also suitable for the angle relationship. The final aim is to get a trade-off between the ideal path (with the least travel time) and the gravity-overcoming path (the spatially optimized path). The rule derives the actual flight route of a missile based on the equation of motion for missile guidance. Reference [17] presented an integrated fuzzy based guidance law to solve the problem of multi-objective optimization in which the final time, energy consumption and miss distance were selected as competing objectives. It also set a fuzzy-based system to acquire the best compromise solution over the trade-off curve. This paper applies the missile guidance law to vehicle path planning. A fixed-step search is performed from the centre - the ideal path-along y -axis until the actual path meeting the path-planning objective is obtained. The actual path is a trade-off between the path-planning of a single vehicle and system optimization.

The ideal path obtained from Section 2.3, which benefits

system optimization, is viewed as the ideal path of system optimization. Because the path for the single vehicle is a time-saving path, the ideal path meeting the path-planning objective of a single vehicle is taken as the ideal path of time, which is extracted using the same algorithm as the ideal path of system optimization except that the degree of congestion at each point in the road network in Formula (1) is

$$m_{fe} = m_c T_c + l_i / v_i \quad (12)$$

The selection criteria of actual path are illustrated below:

A fixed-step search is conducted along y -axis from the centre formed by the two ideal paths, one for system optimization and the other for time saving. The ideal searching area is the overlapped region of two different ideal paths. Actual path planning is performed on all paths in the ideal searching area. With each of the two ideal paths as the benchmark, the path that encloses the minimum area with the ideal path and runs from the origin of the section where the vehicle is located to the destination is selected as the actual path.

In Fig. 5, points A and B are respectively the origin and destination of path planning. The solid-line curve between points A and B is the ideal path of system optimization, and the dotted-line curve between point A and point B is the ideal path of time. The crossing points in the ideal searching area consists of points b , e , d , and g . With the ideal path of system optimization as the benchmark, the actual path enclosing the minimum area with the ideal path is: $AgeB$. With the ideal path of distance as the benchmark, the actual path enclosing the minimum area with the ideal path is: $AgeB$. Path $AgeB$ meets the selection criteria of actual path for the least-time path.

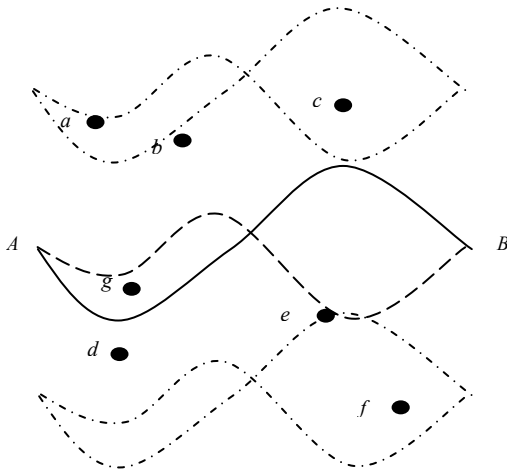


Fig. 5 Schematic diagram of actual path planning

3 Sample calculation and analysis

3.1 Model parameters

The algorithm is implemented in a sample area covering the

road sections as part of a road network consisting of Kehui Road, Kehui South Road, Tatun North Road, and Tatun Road near the Olympic Green in Beijing. Fig. 6 shows the position of the road and the network structure.

In Fig. 6, the traffic network embraces 22 intersections (including the origins and destinations). Before the simulation started, all the intersections and sections in the network are assumed to be free from congestion. Each intersection is indicated with a circle and each section with a dotted line.

To maintain the random and dynamic characteristics of traffic data, the number of vehicles along each section or passing each intersection are random, as well as the destination of the vehicle. To ensure effective performance of the simulation without congesting the traffic network as a whole, the initial number of vehicles should meet two conditions: the distance between any two vehicles is greater than 1m when they are at an intersection of any lane and greater than 6 m when they are along a lane of any section. Traffic state is described using the cell transmission model. The speed limit is 60 km/h for vehicles to pass an intersection and 80km/h for it to run along a section. The signal timing at each intersection is the same as that of the signals in the actual road network.

The proposed method is verified using MATLAB with the simulation time of 1800 s, and it is performed once per second.

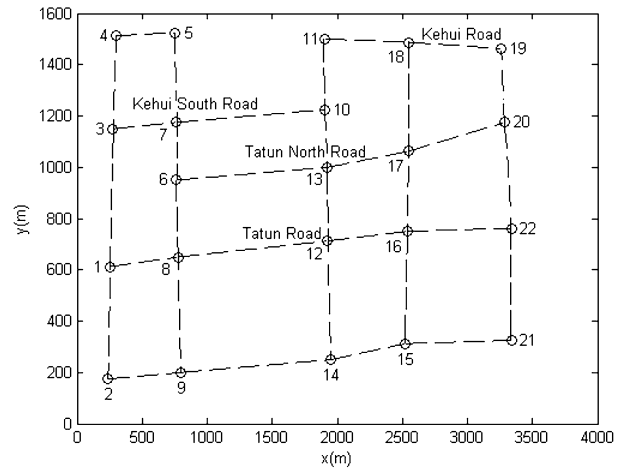


Fig. 6 Road network diagram

3.2 Simulation result and analysis

The degree of congestion of each lane and intersection, as well as the path planning, are all denoted in Fig. 7. All the road sections and intersections other than those on a planned path are respectively indicated with circles and broken lines if they are smooth; or "●" and dash dot lines if they are not smooth; or "×" and "++" if they are congested. Besides the planned path, all the road sections and the intersections that do not need to be indicated are shown as smooth.

To highlight the advantages of the algorithm, the traffic

state is qualitatively expressed. In this paper, traffic state is defined as congested if the average vehicle speed is less than 10 km/h or the number of vehicles passing an intersection is greater than twice of the traffic capacity per signal cycle. The traffic state is defined as unsmooth if the average vehicle speed ranges from 10 km/h to 35 km/h or the number of vehicles crossing an intersection is greater than the traffic capacity per signal cycle but less than twice of the traffic capacity. Traffic state is defined as smooth if the average vehicle velocity exceeds 35 km/h or the number of vehicles passing a crossing below the traffic capacity per signal cycle.

According to Fig. 7, the produced path from the origin (intersection 1) to the destination (intersection 22), is: 1-3-4-5-7-10-11-18-19-20-22. During the running process, intersections 9, 12, 14, 16, and 21 are indicated as unsmooth, sections 3-7, 6-13, 8-12, 9-14, 12-16, 13-17, 14-15, 15-21, and 16-22 are unsmooth, and intersections 8 and 15 and sections 1-8 and 2-9 are congested. All other sections and intersections in Fig. 7 are smooth.

In the path, section 5-7 is not smooth, and all the other intersections and sections are smooth. The fact that the path avoids unsmooth sections and crossings helps with system optimization.

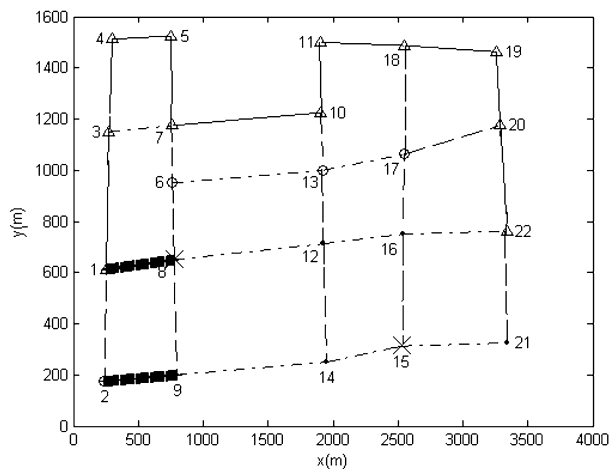


Fig. 7 Simulation result

In this case, the vehicle takes 881.7 s running through the path generated by the algorithm, shorter than the time taken to pass any of the several other paths we selected for simulation, including 1,747.5 s taken to pass the shortest path 1-8-12-16-22, 923.9 s to pass 1-3-7-6-13-17-20-22, and 1,110.2 s to pass 1-3-4-5-7-6-13-17-20-22. This means that the path computed by the algorithm can meet the path-planning objective of a single vehicle with travel time saving.

4 Conclusion

This paper develops a dynamic and real-time single-vehicle path algorithm with full consideration of system optimization. The algorithm settles the conflict between the path-planning

for a single vehicle and for the system optimization, adopting the method of simulating the missile guidance. The ideal path generated by the algorithm assists optimizing the entire traffic system and balancing traffic flow, which also keeps vehicles away from congestion and saves travel time. With the ideal paths in place, the constraints of real road networks and the requirements for path planning of a single vehicle are both considered to select the actual path. Throughout the vehicle's travel, the paths covering the road sections ahead are cyclically revised according to real-time traffic state. This approach also realizes dynamic path planning for a single vehicle. The simulation results show that the algorithm achieved the expected objectives of avoiding congested road sections, saving travel time, balancing traffic flow and optimizing the traffic system. Therefore, it is foreseeable that more vehicles using the algorithm in a traffic network would effectively help with the optimization of the traffic system and balance traffic flow. Nevertheless, further study is still necessary in terms of the path planning algorithm for multiple vehicles.

Acknowledgements

This research was funded by the National Natural Science Foundation of China (No. 50908101).

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